Fingerprint Recognition with Cellular Partitioning and Co-Sinusoidal Triplets

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Abstract: In this fingerprint verification approach, a fingerprint image is divided into equally sized cells and the pattern is represented by a substitute resulting in a feature vector of fixed length. A related ISO standard recommends three different approaches for the selection of these. It suggests a cell size of approximately two ridges per cell. For the co-sinusoidal triplet approach for the retrieval of the spectral component this assumption was investigated. The influence of the cell size on the biometric performance was supported and additionally, a sound comparison method was implemented. To maintain a comprehensible evaluation the open Fingerprint Verification Competition (FVC) databases FVC2000 and FVC2002 were used.

1 Introduction

The biometric characteristic of fingerprints is widely used for verification and identification purposes. Fingerprint recognition became more and more popular through high distribution of fingerprint sensors and the convenience in use. Traditional approaches are based on the extraction of a few stable points (minutiae) that uniquely describe a fingerprint. There are alternative approaches to this method which do not rely on minutiae, like fingerprint correlation [BVG+00] or finger pattern comparison [Hup07]. This paper is focused on fingerprint ridge pattern comparison. The algorithm described is based on the information of the fingerprint ridges and not just singular unique points. There are numerous ways how the pattern is examined. One of the most prominent ones is described in the ISO standard of 2009 [fS06] and will be discussed herein. The approach investigated in this paper has the advantage that features are - unlike finger minutiae data - already in the form of a fixed length feature vector, which is required for further processing such as template protection schemes.
2 Quantized Co-Sinusoidal Triplets

The first step in the generation of a fingerprint template is the conversion of the fingerprint to spectral data. The steps suggested in the ISO standard [FS06] are as follows: Image Preprocessing, Cellular Partitioning and Spectral Component Selection. The image preprocessing step is optional but can have a great effect on the results, since poor image quality leads to missing and spurious features.

2.1 Image Preprocessing

A basic improvement for the problem of strongly varying image qualities that does not consume much processing power is the histogram normalization of the fingerprint image (as for example proposed by Alparslan and Fuatince [AF81]) which already leads to a more consistent image quality. Further enhancements include Gabor filtering [YLJF03], image segmentation [HJ04] or binarization and thinning [Tha03].

2.2 Cellular Partitioning

![Image of fingerprint tessellation]

Figure 1: The tessellation of a fingerprint with an optional X- and Y-Offset.

![Image of fingerprint approximation through co-sinusoidal triplets]

Figure 2: Left: A cell taken from the fingerprint. Right: The approximation through the co-sinusoidal triplets. The $\delta$ parameter is defined as $\delta = d/\lambda \cdot 360$. Source: [FS06]

After image preprocessing, a tessellation of the fingerprint image has to be done. Therefore, the fingerprint will be divided by a grid into cells of size $w \times w$. There is no specification of the optimal size of the cells but in [FS06] it is suggested to have a maximum of approximately two ridges per cell. Therefore, the size depends on the resolution of the sensor. Vannfält and Åström [Vs06] suggested in their thesis a cell size of $5 \times 5$ pixels for a sensor resolution of 250ppi. In case of margins at the borders, an offset can be chosen to get maximum coverage of the fingerprint area. Cells that cannot be fully filled with image information (the rightmost and bottommost cells) will be discarded.
2.3 Spectral Component Selection

There are three different, already established methods to retrieve the spectral information from a cell: Quantized co-sinusoidal triplets (QCT), Discrete Fourier Transformation (DFT) and Gabor filters. There are numerous publications about the use of the DFT (like [Bra89], [Nus82] or [Win76]) as well as the use of Gabor filters (like Yang [YLJF03] or Huppmann [Hup07]) but only the master’s thesis of Vannfält and Åström [Vs06] was concerned with QCT, even though an algorithm using QCT won the FVC of 2000.

QCT are based on the approximation of each finger pattern cell through a cosine triplet \((\theta, \lambda, \delta)\). As seen in Figure 2, the three parameters describe the angle of propagation, the wavelength and the phase. The range of the parameters can be restricted to a minimum and maximum to get the highest variance without repeating structures.

**Angle of propagation** \(\theta\): represents the directional information of a cell. The angle of propagation is measured perpendicular to the crest of the co-sinusoidal function. If the crest is parallel to the vertical axis \(x\), the angle is 0 and it increases with counter-clockwise rotation. The interval \([0, \pi]\) describes all possible orientations of the function. **Wavelength** \(\lambda\): describes the quantity of ridges and the distance between them for one cell. The frequency \(f\) is directly related to this parameter since \(\lambda = \frac{1}{f}\). The range of the frequency is \([0, \text{maximal spatial frequency}]\) where the maximal spatial frequency is the Nyquist frequency [Gre59]. For 2D signal processing the Nyquist frequency is equal to the length of the image diagonal divided by two. **Phase** \(\delta\): describes the distance of the first crest to origin of the cell. It is specified in angular coordinates and therefore is in the interval of \([0, 360]\). It is defined as \(\delta = \frac{d}{\lambda} \cdot 360^\circ\), where \(d\) is the distance. The 2D co-sinusoidal function to approximate the cell is defined as follows:

\[
\text{Cell}_{\theta, \lambda, \delta}(s, t) = \cos(P \cdot 2\pi \cdot f + \delta), \text{ where } P = s \cdot \cos(\theta) - t \cdot \sin(\theta), \text{ and } f = \frac{1}{\lambda} \quad (1)
\]

The parameters \(s\) and \(t\) of the function describe the position of each pixel inside the cell. The valid interval for \(s\) and \(t\) is \(s = [1, \text{image width}]\) and \(t = [1, \text{image height}]\) where \(s, t \in \mathbb{N}\). The resulting values of the function will be quantized and are accurate enough to reconstruct the ridges of the fingerprint depending on the precision for each parameter. The amount of possible values of the quantization (bit-depth) depends highly on the cell size. The smaller the cells, the less information is necessary to approximate a cell adequately. Therefore, it is required to find a suitable bit-depth depending on the resolution of the cells (see 3 for an example). The substitution through the function automatically leads to a tolerance for errors in each cell (noise in the fingerprint image) and therefore it is very important to correctly estimate the bit depth.

To select the most suitable triplet to approximate a cell, the following approach is chosen. First a normalization of the fingerprint values to the range \([-1, 1]\) is done. Then, the distance between the fingerprint cell and all possible synthetic cells (candidates) is calculated and the synthetic cell structure with the minimum distance is used to represent the information of the cell. Here, the Euclidean distance function is used to determine the resemblance of two cells, even though different distance functions, like hamming distance can also be used. In the case that there are more than one cell with the same distance, the
following prioritization shall be employed. The triplet with the lowest frequency ($\delta$) has the highest priority, then the triplet with the highest wavelength and finally the triplet with the lowest angle of propagation.

The number of the possible candidates depends on the bit-depth for the parameters and is $2^{l+m+n}$ where $l$, $m$ and $n$ are the the bit depths for $\theta$, $\lambda$, and $\delta$. The specific values are defined through an equidistant distribution between zero and the respective maximum values. For $\theta$ the maximum is 180, for $\lambda$ the maximum is the Nyquist frequency and for $\delta$ the maximum is 360.

### 3 Experiments

In order to investigate the influence of the cell size, the False Acceptance Rate (FAR), the False Reject Rate (FRR) and the Equal Error Rate (EER) for cell sizes in the range of $5 \times 5$ pixels up to $18 \times 18$ pixels were studied. The comparison algorithm used is based on the similarity of the cell triplets in the reference and the probe. Each cell triplet of the probe will be compared to the corresponding cell triplet in the reference. A comparison score will be calculated depending on the similarity of all cell pairs. With an increasing score the probe resembles the reference better, therefore it is a similarity score.

In order to allow a positive comparison when noise is present in the images, a certain difference is acceptable. This is taken into account by matching cells only if their similarity is above a certain threshold. One problem that occurs is that occasionally, some cells score high enough to be above the threshold even though the surrounding cells do not match. In order to reduce the errors introduced by these outliers, the neighborhood around the current cell is taken into account by giving it a higher score if the surrounding cells match as well.

All possible combinations of the three parameters were considered as well during the tests but did not lead to better results. The test images were preprocessed by the VeriFinger SDK 6.0. Different image enhancement filters followed by a binarization were applied.
and the orientations and positions of the cores were extracted. The images were then
aligned on the cores to overcome translation and orientation problems. Images with no
cores present and images that VeriFinger could not process were excluded from the test.
Thus, the fingerprint set was reduced by 18% to a total of 656 prints from originally 800
for FVC2000 DB2_b and by 8.75% to a total of 730 of 800 prints for FVC2002 DB2_b.
The chosen bit-depth for the angle of propagation $\theta$ was 5, for the wavelength $\lambda$ 4 and for
the phase $\delta$ 5.

4 Results

After conducting an evaluation on the dimensions for each parameter of the quantized co-
sinusoidal triplet, the following results were discovered. The acquisition is based on the
FVC2000 DB1_A and FVC2002 DB2_A. The results show the Equal Error Rates (EER)
for each cell size. As can be seen in Table 1, the cell size that results in the least average
error rate and therefore the optimal cell size for the FVC dataset DB1_a is 16.

<table>
<thead>
<tr>
<th>Cell Size (in Pixels)</th>
<th>$\theta$</th>
<th>$\lambda$</th>
<th>$\delta$</th>
<th>Mean (for $\theta$, $\lambda$ and $\delta$)</th>
</tr>
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<tbody>
<tr>
<td>5</td>
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<tr>
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<td>9</td>
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<td>0.1482</td>
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<td>0.2231</td>
<td>0.2558</td>
<td>0.2849</td>
<td>0.2546</td>
</tr>
</tbody>
</table>

Table 1: The EER for $\theta$, $\lambda$, $\delta$ and the mean for all three parameters

5 Conclusion

After conducting the evaluation of the different cell sizes using the quantized co-sinusoidal
triplet approach, the assertion of the ISO standard [fS06] - to have approximately two
ridges per cell - could be experimentally validated. The minimum possible ridge frequency
is zero, which is present when a cell is of homogeneous intensity. This was observed very
frequently with small cell sizes (around $5 \times 5$ to $6 \times 6$ pixels). It implies that the suggested
average of two ridges per cell is violated. Large cell sizes lead to a rougher approximation

of the actual content of the fingerprint image and make the method less error prone. When looking at the results in Table 1, the cell sizes between $11 \times 11$ and $14 \times 14$ produce stable equal error rates at a stable level. The maximum ridge frequency for those cells sizes is 3 (defined by the Nyquist frequency). The optimal solution resulting in the best performance considering the EER was achieved with a size of $14 \times 14$ for the combination of all three parameters. A cross-check with the database FVC2000 DB_1a and FVC2002 DB_2a with different sensor properties shows, that the cell size has to be adapted specifically for the available image data.

References


114