Tracking Algorithms for Bistatic Sonar Systems

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Abstract: An active sonar system consists of a sound source activating a surveillance area and a receiver listening for echoes reflected from targets. In a bistatic setup, the source and the receiver are not collocated. The resulting measurement model is described by a non-linear function of the target state, the given bistatic geometry and environmental parameters. For target tracking it is mandatory to have an adequate description of the uncertainty on the target state that is resulting from the uncertainties in the measurements and environmental parameters. An inadequate description would decrease the performance of the fusion process. To cope with the nonlinearity in the measurement model, four different methods can easily be implemented based on the Kalman Filter framework: linear transformation of each measurement in Cartesian coordinates with tracking in the Cartesian system, unscented transformation of each measurement in Cartesian coordinates with tracking in the Cartesian system, extended Kalman filtering and unscented Kalman filtering. In this paper, we compare these methods by simulating their performance in an operationally relevant setup.

1 Introduction

Bistatic sonar has been identified to improve surveillance performance compared to monostatic systems, in particular when exploiting the operational benefit of having a covert receiver. In this setup, the target is losing the advantage of a stealth design. Operational concepts extend this idea also to the 'multistatic setup' consisting of multiple sources and receivers. However, due to e.g. communication constraints within the multistatic setup, the pure bistatic scenario has also there an important role: Tracking of contacts from a bistatic detection opportunity might lead to tracks which are then sent to a track fusion centre. The fusion centre collects bistatic tracks from all participating receivers in the multistatic setup. Each bistatic track should be as accurate as possible to ensure at the fusion centre a correct association of received bistatic track information to the same target [GCCG06]. From a system design point of view, the accuracy of measured contact and environmental data is limited by the given variability of the underwater sound channel and by budget or feasibility constraints on the quality and number of measurements. By
precise modelling and calibrating the underwater sound channel the quality of a bistatic setup can be improved [Cox89]. But in spite of appropriate corrections, some parameters in the multistatic environment may remain imprecise. Thus, from a sequential tracking point of view, it is important to correctly model the uncertainties accompanying the actual measured values, otherwise the tracking filters are not able to fully exploit the presented data.

For bistatic sonar the measurements are described by a non-linear function of the target state, the given bistatic geometry and environmental parameters (like the speed of sound, the source and receiver positions and the receiver heading). The measurement model includes noise describing the uncertainty inherent in the measurement process for all arguments of this non-linear function. In order to apply sequential tracking techniques in the framework of a state-space Kalman filter, approximations of the non-linearity and/or its effect on associated uncertainties have to be applied. In [Cor06], measurement uncertainties are transformed into Cartesian coordinates by linearizing. We will compare this approach with an alternative strategy based on the Unscented Transform (UT) [JU04]. After the transformation, a linear Kalman filter can be used to track contacts. Two alternative procedures are available, known as extended Kalman Filter (EKF) [DP66] and unscented Kalman Filter (UKF) [JU04] that can be adapted to account for uncertainties in the environmental parameters.

In this paper, we present a performance analysis for these four different methods in terms of Monte Carlo simulations for an operationally relevant bistatic setup, together with a comparison of their tracking results to the Cramer Rao Lower Bound (CRLB) of the associated estimation problem.

This paper is organized as follows: In the next section the measurement model is described. In section 3 we address the transformation of measurements into Cartesian coordinates considering additional imprecision in the environmental parameters. Different approaches for Kalman Filtering will be considered in section 4 and finally we discuss the results of Monte Carlo runs in section 5.

## 2 Measurement model

The measurement is described as a non-linear function of the target state.

\[ z = h(x) + w, \]  

with Gaussian measurement noise \( w \). The measurement vector has two components: azimuth angle \( \varphi \) and time of arrival \( \tau \). Let \( p = (x, y)^T \) be the target position and \( s = (s_x, s_y)^T \) and \( o = (o_x, o_y)^T \) the position of the source and the receiver respectively. The receiver orientation (heading) is given by \( \vartheta \), c.f. Fig. 1; then the measurement function can be expressed as

\[
\varphi = \arctan \left( \frac{x - o_x}{y - o_y} \right) - \vartheta, \\
\tau = \left( \| p - s \| + \| p - o \| \right) / c_s,
\]  

(2)
where $| \cdots |$ denotes the Euclidian norm and $c_S$ is the propagation speed of sound in water.

Figure 1: Bistatic Setup; one source, one observer and a single target

3 Mapping of Uncertainties in the Measurements to Cartesian Coordinates

According to (2) and Fig. 1, to transform measurement values into $x/y$ coordinates, sound speed $c_S$, receiver $o$ and source $s$ positions and receiver orientation (heading) $\vartheta$ have to be known. In fact, only estimates of these values will be available. Hence, we model the uncertainty following [BDK08] by

- $o \sim \mathcal{N}(o; \bar{o}, P_O)$
- $s \sim \mathcal{N}(s; \bar{s}, P_S)$
- $c_S \sim \mathcal{N}(c_S; \bar{c}_S, \sigma_{c_S})$.

An error in the receiver heading can easily be incorporated by enlarging the error in azimuth information, i.e. if $\sigma_\varphi$ is the deviation of the measurement error and if $\sigma_\vartheta$ describes the uncertainty in the receiver heading, the actual error of the bearing estimate is enlarged by $\sigma_\hat{\varphi} = \sqrt{\sigma_\varphi^2 + \sigma_\vartheta^2}$. Noting that the extension to the more sophisticated case is straightforward we set the expected receiver heading to $0^\circ$ in this paper.

This results in a new definition of an artificial measurement vector

$$z_a = (\tau, \varphi, c_S, s^T, o^T).$$

(3)

The 2D-target position $x = (x, y)$ can finally be calculated from

$$x = g(z_a),$$

(4)
where we use the following equations:

\[ BL = s - o \]
\[ \alpha = \arctan(BL_x / BL_y) - \varphi \]
\[ \Delta = \sqrt{BL_x^2 + BL_y^2} \]
\[ \text{range} = \frac{(\tau \cdot cS)^2 - \Delta^2}{2(\tau \cdot cS - \Delta \cos(\alpha))} \]
\[ x = \sin(\varphi) \cdot \text{range} + o^{(1)} \]
\[ y = \cos(\varphi) \cdot \text{range} + o^{(2)} \]

In [Cor06] a linearization approach has been presented to derive the Cartesian covariance matrix: \( g \) is approximated by linearizing. We compare this approach to the Unscented Transform (UT) [JU04]. The UT delivers the corresponding target state estimate and covariance matrix by exploiting the relationship given in \( g \). Thus, it provides a very simple method to incorporate additional errors.

### 4 Implementation of the Kalman Filter Scheme

We are interested in investigating effects of additional error sources on Kalman Filtering. The target position estimate derived in the previous section will help us to destine the track initialization point. In addition to the target position we initialize target velocities with zero mean and deviation of 5m/s in each component.

The Kalman Filter Scheme consists of two steps: Prediction of the track state (usually in Cartesian coordinates) and track update using the measurement information. Two different ways to handle the non-linear measurement equation in these schemes are investigated: (i) The Kalman Filter updates the target state in Cartesian coordinates (Cartesian KF) (ii) The predicted state is transformed into the measurement space to perform the filter update (UKF/EKF). Fig. 2 is illustrating these two approaches.

![Track update schemes: Cartesian KF (left); Transformation of measurement \( z_k \) into Cartesian state; UKF/EKF (right): transformation of predicted track state into measurement space](image)

The application of the Cartesian Kalman Filter works straightforward by exploiting the transformation described in the previous section. Instead, the standard UKF and EKF do
not care about additional error sources by default, but can easily be transferred by defining
the vector \( \mathbf{a} = (c_S, \mathbf{o}^T, \mathbf{s}^T) \), containing the environmental parameters. As in the previous
section we can assume the heading vector to be known and pick up the uncertainty in the
azimuth uncertainty.

The aim of target tracking is to destine the conditional probability of the target state \( \mathbf{x}_k \)
at time \( t_k \) given the measurement history \( Z^k = \{ \mathbf{z}_1, \ldots, \mathbf{z}_k \} \) and the a priori knowledge
about the environmental parameters, i.e. \( \mathbf{a} \sim \mathcal{N}(\mathbf{a}; \bar{\mathbf{a}}, \mathbf{C}_a) \):

\[
p(\mathbf{x}_k | Z^k, \mathbf{a}) \propto p(\mathbf{z}_k | \mathbf{a}, \mathbf{x}_k) p(\mathbf{a} | \mathbf{x}_k) p(\mathbf{x}_k | Z^{k-1})
= \mathcal{N}(\mathbf{z}_k; h_{C_{\rightarrow M}}(\mathbf{x}_k, \mathbf{a}), \mathbf{R}) \mathcal{N}(\mathbf{a}; \bar{\mathbf{a}}, \mathbf{C}_a) \mathcal{N}(\mathbf{x}_k; \mathbf{x}_k | k-1, \mathbf{P}_{k|k-1})
= \mathcal{N}(\mathbf{z}_k; h_{C_{\rightarrow M}}(\mathbf{x}_k, \mathbf{a}), \mathbf{R}) \mathcal{N}\left( \begin{pmatrix} \mathbf{x}_k \\ \mathbf{a} \end{pmatrix}; \begin{pmatrix} \mathbf{x}_k|k-1 \\ \bar{\mathbf{a}} \end{pmatrix}, \begin{pmatrix} \mathbf{P}_{k|k-1} & \mathbf{O} \\ \mathbf{O} & \mathbf{C}_a \end{pmatrix} \right) \tag{6} \]

The proportionality in the first line holds due to the Bayes rule. Additionally, we exploit
that the measurements do not depend on the measurement history if the target state is
known. According to our assumptions the probability densities can be described by Gauss-
ians (line 2), whereas the additional error vector \( \mathbf{a} \) is independent of the target state. In
the third line we combine the probability densities regarding target state and system pa-
rameter information, i.e. the target state vector is artificially extended,

\[
\mathbf{x}_a = (\mathbf{x}_T, \mathbf{a}_T)^T.
\]

Using linearization of the relation given by \( h_{C_{\rightarrow M}}(\mathbf{x}_k, \mathbf{a}) \approx \mathbf{H}_1 \mathbf{x}_k + \mathbf{H}_2 \mathbf{a} + \mathbf{b} \) we can
exploit the product formula for Gaussian densities and derive the following update formu-
las:

\[
\begin{align*}
\mathbf{x}_{k|k} &= \mathbf{x}_{k|k-1} + \mathbf{W}(\mathbf{z}_k - h(\mathbf{a}, \mathbf{x}_k)) \\
\mathbf{P}_{k|k} &= \mathbf{P}_{k|k-1} - \mathbf{W} \mathbf{S} \mathbf{W}^T,
\end{align*}
\]

where \( \mathbf{S} = \mathbf{H}_1 \mathbf{P}_{k|k-1} \mathbf{H}_1^T + \mathbf{H}_2 \mathbf{C}_a \mathbf{H}_2^T + \mathbf{R} \) and \( \mathbf{W} = \mathbf{H}_1 \mathbf{P}_{k|k-1} \mathbf{S}^{-1} \). The update
formulas are very similar to the standard EKF formulas; we abstain from updating the
information in \( \mathbf{a} \), since it is assumed to be independent for different pings. We can easily
replace the linearization that was used in the derivation by the UT to approximate \( \mathbf{S} \) and
\( \mathbf{W} \).

5 Numerical Results of Cartesian Transformation

5.1 Simulation Setup

For numerical evaluation we consider a scenario where the receiver is located at
\((-1 \text{km}, 0 \text{km})^T\) and the source is at \((1 \text{km}, 0 \text{km})^T\). Measurement errors are chosen to be
\( \sigma_\varphi = 2^\circ, \sigma_\vartheta = 2^\circ, \sigma_r = 0.001 \text{s} \) and \( \sigma_{c_S} = 2 \text{m/s} \). Additionally we assume equal and
uncorrelated errors for the components of source and receiver position with \( \sigma_L = 20 \text{m} \),
i.e. \( P_O = P_S = \text{diag}(\sigma^2_m, \sigma^2_n) \). For this setup the Cartesian transformations (Linearization/UT) are compared: Results will be presented and discussed in Section 5.2.

In order to compare the different approaches to implement the Kalman Filter scheme (see section 4), the simulation setup described above is extended in the following way: For each Monte Carlo run a target is inserted at \((2\text{km}, 2\text{km})^T\). Its constant velocity is sampled from a Gaussian distribution with zero mean and deviation of 5m/s in \(\dot{x}\) and \(\dot{y}\) (Targets that cross the line between source and receiver are ignored).

In the remainder of this subsection we describe used measurements of performance: Average Estimation Error, Cramer-Rao Lower Bound and Consistency.

### 5.1.1 Average Estimation Error

The root-mean-square error of the position estimate (RMSPOS), is an absolute error measure and direct performance criterion. It is averaged over all simulation runs. The RMS error from \(N\) Monte Carlo runs for the position estimates \(\tilde{x}\) and for truth \(x\) is

\[
\text{RMS}(x) = \sqrt{\frac{1}{N} \sum_{i=1}^{N} |\tilde{x}_i - x|^2}
\]  

(8)

### 5.1.2 Cramer-Rao Lower Bound

Since we consider non-linear measurements and additive white Gaussian noise \(w\), the Cramer-Rao lower bound (CRLB) can be derived in a standard way. The general calculation of the Fisher information matrix can be replaced by a more specialized formula,

\[
J_0 = \mathbb{E} \left\{ \left[ \nabla_{x_a} \log \Lambda(x_a) \right] \left[ \nabla_{x_a} \log \Lambda(x_a) \right]^T \right\} = \frac{\partial h}{\partial x_a} \text{Cov}(w)^{-1} \frac{\partial h}{\partial x_a}^T,
\]  

(9)

where \(\Lambda(x_a) = p(z|x_a)\) is the likelihood function. In our application we consider the 9-dimensional state vector \(x_a\) versus the 2-dimensional measurement vector \(z_a\), so the matrix \(J_0\) will not be invertible. This shows that we cannot estimate the full state vector \(x\) without additional assumptions. As information is additive, these additional assumptions in form of a prior distribution in \(a\), can be added to the Fisher information matrix [Van68],

\[
J = J_0 + J_P
\]

(10)

where \(J_P\) is the Fisher information of the prior:

\[
J_P = \begin{bmatrix} 0 & 0 \\ 0 & C_a^{-1} \end{bmatrix}
\]

(11)
5.1.3 Consistency

Filter consistency is usually measured using the normalized (state) estimation error squared (NEES), defined as

\[ \epsilon = \tilde{x}^T \mathbf{P}^{-1} \tilde{x}, \]

which should be chi-square distributed with \( \eta_x \) degrees of freedom, if the filter is consistent. In Monte Carlo simulations that provide \( N \) independent samples \( \epsilon_i, i = 1, \ldots, N \), the average NEES is

\[ \bar{\epsilon} = \frac{1}{N} \sum_{i=1}^{N} \epsilon_i. \]

It has to be tested whether \( N \bar{\epsilon} \) is chi-square distributed with \( N\eta_x \) degrees of freedom. This hypothesis is accepted, if \( N \bar{\epsilon} \) is in the appropriate acceptance region.

5.2 Comparing Linearization and Unscented Transform

In Fig. 3 performances of the two transformation approaches (Linearization / UT) are compared with respect to the evaluation scenario described in subsection 5.1. We run \( 10^3 \) Monte Carlo runs and calculate the target position and covariance in a grid of a 10km \( \times \) 10km region. The two approaches are analyzed by the Root mean squared error of the position estimate (RMSPOS) and the normalized estimation error squared (NEES) as a measure of consistency. The NEES are calculated for the 2-dimensional position estimate only, thus the averaged NEES should be close to 2. Obviously, the method based on linearization tends to optimistic estimation behaviours (NEES are too high, see Fig. 3c) whereas the method based on UT produces slightly to high covariances (NEES are too low in some region, see Fig. 3d). The reason for the optimistic behaviour of linearization is that the covariances are approximately equal to the Cramer Rao Lower Bounds that can not always be achieved. Otherwise the covariances of the UT depend on the actual diffusion of the sigma-points. Besides a large diffusion of sigma-points can effect estimation accuracy, this reflects in slightly worse estimation performance near to the base line of source and receiver, see Fig. 3b.

5.3 Comparing Implementations of the Kalman Filter Scheme

To analyze the performance of the different types of Kalman Filter we run \( 10^4 \) Monte Carlo runs. Each target starts at position (2km, 2km)\( ^T \) and is moving with constant velocity that is sampled from a Gaussian density with zero mean and deviation of 5m/s in \( \dot{x} \) and \( \dot{y} \). Targets that the cross the line between source and receiver are ignored. Results of the RMSPOS for the four different tracking approaches are illustrated in Fig. 4-5 and are compared to the Cramer Rao Lower Bound. Fig. 4(a) corresponds with the standard settings in the parameter uncertainties and a measurement update rate of 60 seconds, i.e. time between two consecutive pings is 60 seconds. The two approaches based on the
unscented transform show comparable good results. Surprising is the poor performance of the EKF. Also shown in Fig. 4 are additional tracking results with slightly modified parameters. On the right hand side a short update rate of 10sec is investigated. Obviously, the problem of the EKF seems to disappear and the EKF produces comparable good results to the UKF. In Fig. 4(c-d) the influence of the accuracy in source and receiver position is considered. Fig. 5 corresponds to the same scenario, but small azimuth and heading errors ($\sigma_\phi = \sigma_\theta = 1^\circ$). Again we note that for large update intervals (left side) the EKF shows worse performance than the other approaches; but performance is improved compared with the results shown in Fig. 4.

The observed effects are summarized in the following statements:

- tracking performance is affected by the measurement and system parameters uncertainty: If the total error is strongly bearing dependent, i.e. large bearing errors or high precision in range, the methods based on UT show superior performance than the methods based on linearization.
Figure 4: Comparison of different types of Kalman Filters. In (a) the standard setting was used; (b-d) illustrate results for modifications in bearing and sensor precision, as well as in ping update rate.

- tracking performance depends on the measurement update rate: The UKF and the EKF show worse performance than the Cartesian Kalman Filters during the first pings. A long inter-ping period may cause divergence of the EKF.

These effects are further examined in the next two subsections.

5.3.1 Dependencies on the Measurement and System uncertainties

We consider again a target at position \((2\text{km},2\text{km})^T\). The Cartesian KF and EKF/UKF use approximations of the mapping of measurements to Cartesian and reverse. In Fig. 6(a,c) the grey dots denote Cartesian representatives when sampling from the (extended) artificial measurement vector. The red and blue ellipses show the $3\sigma$-gate of the approximated Gaussians generated by linearization or UT respectively. In contrast, on the right hand side (Fig. 6(b, d)) the grey dots denote measurement representatives that is sampled from the extended state vector, whereas the uncertainty in the Cartesian position is given by the
Figure 5: Comparison of different types of Kalman Filters. (a-d) illustrates results for modifications in bearing and sensor precision, as well as in ping update rate.

CRLB. Again, red and blue denote the approximation resulting from the linearization or UT method. Fig. 6(a-b) refers to the standard parameter settings, see subsection 5.1; in Fig. 6(c-d) more precise source and receiver positions are simulated (σ_L = 5m). For the standard scenario (Fig. 6(a-b)) both methods (UT and Linearization) deliver quite good approximations of the actual densities. As noted before, the covariances given by the UT are slightly higher than of the linearization method. When improving accuracy in source and receiver positions, the localization error becomes more and more bearing dependent. In Cartesian this is shown by a banana shape, but also in the measurement space the actual density differs strongly from a Gaussian shape and both approximation schemes (either UT or linearization) give non satisfying results. However the UT delivers larger covariances which seem to fit better and be more robust.
5.3.2 Dependency on the Measurement Update Time

For enlarging time between two consecutive pings the uncertainty in the predicted state increases, especially in the beginning when there is only poor knowledge about the target velocity. During the EKF update step the Jacobian is evaluated at the predicted state. Since we are faced with a large prediction covariance matrix and compared to this a high measurement precision, linearization according to the Jacobian gives only a poor performance in the region it is of actual interest. In Fig. 7 results of a UKF and EKF update step are compared relative to the measurement uncertainty in time and azimuth (black). The blue and red ellipses show the measurement representative of the updated track state for the UKF and the EKF, respectively. Since the updated track state is only available in Cartesian coordinates during the UKF and the EKF update, it needs an additional transformation to be illustrated in the measurement space. The black ellipse shows the current measurement information together with measurement uncertainties in $\phi$ and $\tau$, that has been used for track update. As is seen in the figure the EKF covariance (red) is rotated compared to the actual uncertainty. This can be explained by a failure of linearization.
\( \sigma_L = 20 \text{m}, \sigma_\phi = \sigma_\theta = 1^\circ, T = 60 \text{s} \)

Figure 7: true measurement (black); transformed update: UKF (blue), EKF (red), (green) covariance according to the Jacobian that was used in the EKF

The EKF update is done according to the Jacobian that is evaluated at the predicted track state. For transforming the updated track state back into measurement space we evaluate the Jacobian at the updated track state. A significant difference between predicted and updated state (that is likely for long inter-ping periods) results in a rotation of the covariances. This is demonstrated by the green ellipse that represents again the transformation of the updated track state, but here the Jacobian is evaluated at the predicted state. This ellipse looks much more reasonable, but it will not be considered in the further tracking steps. Obviously, the linearization delivers only an unsatisfactory approximation that can result in divergence of the EKF. In contrast, the covariance of the UKF (red) is very large and compensates for approximation errors. Both approaches UKF and EKF will not show optimal performance in such situations. The EKF may diverge or be biased, whereas the results of the UKF are degraded by large covariances. As shown in the tracking results, the UKF is more likely to stabilize when the accuracy in the state estimate is improved after some pings.

6 Conclusion

Due to the nonlinearity in the measurement model approximation techniques are necessary to apply sequential target tracking to data from a bistatic sonar system. In this paper,
we have implemented two approximation techniques, i.e. linearization and the Unscented Transform, and studied their performance when mapping bistatic data to the Cartesian system. Special emphasis lies on incorporating uncertainties inherent in the environmental parameters. Tracking is possible for both techniques in the Cartesian system. Using also the Extended Kalman filter and Unscented Kalman filter schemes, four different tracking methods are available. We compared their performance based on simulated data, relative to the CRLB. Comparing the approximation techniques linearization and Unscented Transform, we found that linearization tends to underestimate the actual errors whilst the Unscented Transform tends to overestimate. Referring to tracking performance, UT seems to be preferable. During the course of the tracking process the accuracy of the target state estimation is increased (given that the target motion model fits), whereas the accuracy of the measurements remains constant (fixed measurement errors). Since the performance of the approximation seems to depend on the accuracy available in the coordinate system before transformation, we propose to switch between approximation techniques: Starting with tracking in Cartesian coordinates at initialisation phase of a track and then switching to an UKF (or EKF) scheme.

References


