Analytical data modeling of investment project financing process

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Abstract: The present work is devoted to the research of investment projects’ financing issues. Within this paper a data analytical tool for an optimal financing schema computation on the basis of solving the multicriterion dynamical problem is being presented. The model reflects the interests of both company and investor receiving maximum profit from the project execution as well as selection of a priority source of financing on the basis of capital cost minimization principle. We have developed the software on Java in order to perform an automated computation of an optimal financing schema using the suggested algorithm. The developed software is a complete solution for working with project data, obtaining the results (in the text and chart views) and generating reports.

1 Introduction

Professional project management has recently been associated with creative activities in management and coordination of human and material resources. Within a project lifecycle the application of contemporary management methods and technologies helps to attain project goals in scope, costs, timely and high-quality execution of works, meeting the expectations of all project participants [Vo95].

Within the frames of project management both engineering, investment projects and projects of social importance, economic, organizational projects, etc. are successfully executed.

The object of the present study are investment projects differing in the fact that already at commercial offer stage the purpose, expenses, duration and deadline of the project, necessary resources. Actual costs of the project depend first of all on realization of project works, required assets must be supplied according to a prescribed timetable [Di00]. In spite of the fact that the general realization scheme for investment project has been thoroughly studied, there are a number of problems needing solution in planning of each particular project.
In contemporary project management practice the problem of planning and ensuring of project financing is especially important (fig. 1). This is due to the fact that often project budget exceeds many times the company’s own assets, therefore, the only way to ensure adequate financing is the attraction of capital.

This work treats profit-oriented investment projects. In case a project is fully financed by a company, it will receive all profits and spend them as it likes. The situation is different when an external investor (hereinafter investor) participates in project financing. Both investor and company are interested to obtain profit from project execution. Then a question arises: how to distribute profit between those parties?

![Diagram of investment project financing problems]

The simplest and most often applied approach to the problem solution is an agreement with investor to apportion him either a fixed amount of money or the profit is distributed proportionately to invested capital.

2 Mathematical model of financing process

This work dwells on the problem of project financing schedule formation on the basis of averaged profit maximization criteria for both parties. In other words, the shares of investor and company in financing of various stages of investment project must be calculated. Lump-sum investments aren’t taken up in this work; the financing is carried out during the phases of the project execution (investment term is less than project
execution duration). In order to solve this problem, there is proposed to develop a mathematical model of the project financing process.

The purpose of modeling is determination of optimal values of the following parameters for each stage of project execution:

- scope of external investments. If investments are not necessary at a certain stage, this scope will be zero;
- share of income from project realization apportioned to investor;
- share of income from project realization apportioned to company;
- share of company income to be invested at the next stage of project.

Accordingly such optimality criteria may be separated for the project financing schedule:

- Maximization of investor’s total profit. Total profit of investor equals his total income from project execution less his total investments;
- Maximization of company’s total profit from project realization. Company’s expenses consist of two components: company’s direct investment into project execution and apportionment from its income for financing of the next stage;
- Minimization of project execution capital cost. As the project is financed from two sources (company’s assets and investor’s assets), the priority source may be selected on the basis of evaluation of financing source capital cost. Thus, company’s capital cost may be evaluated with return coefficient of an alternative investment, whereas investor’s capital cost by minimum value of this project profitability.

Let us assume that all resources necessary for the project execution are presented in monetary value and the whole period of project execution is divided into \( N \) equal stages. For each stage we know the scope of assets \( d_k (k = 1, N) \) necessary for investment (expenses), and income from the project \( p_k (k = 1, N) \) expected at each stage. In this work the values of \( d_k \) and \( p_k \) are treated as determined and the risk of failure is not accounted for.

The main requirement to project realization is the necessity of its continuous execution in accordance with the preplanned schedule of works. In the mathematical model this requirement is expressed by the fact that at the beginning of each stage the apportioned money resources are sufficient to cover expenses at this stage. To this end we can apply:

- \( q_k (k = 1, N) \) - Assets apportioned by the company directly to project execution. The scope of apportionment is planned by the company for each stage and thus fixed;
- \( x_k (k = 1, N) \) - External investment with its scope variable for each stage; to be determined in the process of problem solution. The value of \( x_k \) may be limited with a certain maximum value \( x_k^{\text{max}} \);
- \( p_k^s (k = 1, N) \) - Scope of internal project investment as apportioned by company from its income of \( k \)-th stage for \((k+1)\)-th stage. This is also a variable value.
The input and output data is represented on the fig. 2 in diagram form.

Thus, to ensure continuous financing of project following condition must be met:

\[ x_k + p_{k-1}^I = d_k - q_k \quad (k = 1, N). \]  

(1)

Company and investor obtain income from results? of project execution \( p_k \ (k = 1, N) \).

Let us present the results? at \( k \)-th stage as a sum of two components: investor’s \( p_k^I \) and company’s income \( p_k^E \).

\[ p_k^I + p_k^E = p_k \quad (k = 1, N). \]  

(2)

---

**Fig. 2 – Input and output data of the problem**
The company may invest a part of its income at $k$-th stage into the next $(k+1)$-th stage of project, thus reducing external investment. Let us denote the amount of apportionment as $p_k^S$, with

$$p_k^S \leq p_k^E \ (k = 1, N).$$

The feasibility of self-investment at a certain stage will be determined on the basis of the capital cost minimization criterion:

$$V = \sum_{k=1}^{N} \frac{c^I x_k + c^E (p_{k-1}^E + q_k)}{(1+r)^k} \rightarrow \min,$$

where $c^I$ – coefficient characterizes the cost of external investment for the company; $c^E$ – coefficient characterizes the cost of company’s own capital; $r$ – market value of capital.

Both company and investor strive to maximize their profit from project execution. We shall calculate the profit on the basis of net present value (NPV).

Investor’s thrive for maximum profit may be presented as such a mathematical criterion:

$$NPV^I = \sum_{k=1}^{N} \frac{p_k^I - x_k}{(1+r)^k} \rightarrow \max,$$

where $x_k$ – scope of investment at $k$-th stage (equals zero in case of none).

Company’s interests may be presented as:

$$NPV^E = \sum_{k=1}^{N} \frac{p_k^E - p_{k-1}^E - q_k}{(1+r)^k} \rightarrow \max.$$

As a result we have an investment model consisting of criteria (4)-(6) under limitations (1)-(3), as well as

$$x_k \leq x_{\max} \ (k = 1, N);$$

$$p_k^I \geq 0, \ p_k^E \geq 0, \ p_k^S \geq 0, \ x_k \geq 0 \ (k = 1, N),$$

with $p_0^S = 0, \ p_0^E = 0$.

Then the problem consists in finding such $x = \{x_k\}, \ p^I = \{p_k^I\}, \ p^E = \{p_k^E\}, \ p^S = \{p_k^S\}$, that meets limitations (1)-(3), (7)-(8) and give extreme to functions (4)-(6).
3 Algorithm of optimization - problem solution

This problem belongs to multicriterial linear programming class with continuous variables. For its solution we must perform a weighted convolution of criteria (4)-(6) after preliminary normalization. Criteria are normalized using a monotonous transformation of type [MV82]:

$$
\omega_i(f_i(\alpha)) = \begin{cases} 
\frac{f_i^0 - f_i(\alpha)}{f_i^0 - f_i((\text{min}))}, & \forall i \in I_1 \\
\frac{f_i(\alpha) - f_i^0}{f_i((\text{max})) - f_i^0}, & \forall i \in I_2
\end{cases}, \tag{9}
$$

where $f_i((\text{min}))$, $f_i((\text{max}))$ — respectively highest (lowest) values of maximized (minimized) target functions on a plurality of acceptable alternatives; $f_i^0$ — optimal value of $i$-th target function on a plurality of acceptable alternatives; $I_1, I_2$ — plurality of indices for respectively maximized and minimized target functions.

Let us denote normalized criteria for each stage as respectively $\overline{NPV}_k^l$, $\overline{NPV}_k^E$ and $\overline{V}_k$, then the weighted convolution $F_k$ takes the form:

$$
F_k = \alpha^l \cdot \overline{NPV}_k^l + \alpha^E \cdot \overline{NPV}_k^E + \alpha^V \cdot \overline{V}_k \rightarrow \min
$$

$$
\alpha^l + \alpha^E + \alpha^V = 1, \tag{10}
$$

$$
\alpha^l \geq 0, \alpha^E \geq 0, \alpha^V \geq 0,
$$

where $\alpha^l$, $\alpha^E$ and $\alpha^V$ characterize the degree of importance of criteria $NPV^l$, $NPV^E$ and $V$ respectively. From economics’ point of view of the larger value $\alpha^l$ ($\alpha^E$) corresponds to the priority of investor (company) in distributing the profit of the project.

Transformation (9) being linear, the total criterion, $F$ is additive, with

$$
F = \sum_{k=1}^{N} F_k \rightarrow \min
$$

$$
F_k = F_k(p_k^l, p_k^E, p_{k-1}^E, x_k) = \alpha^l \cdot \overline{NPV}_k^l(p_k^l, x_k) + \\
+ \alpha^E \cdot \overline{NPV}_k^E(p_k^E, p_{k-1}^E) + \alpha^V \cdot V_k(p_{k-1}^E, x_k). \tag{11}
$$

Thus, the initial problem may be divided into $N$ sub-problems, the problems solved at $(k-1)$-th and $k$-th stages being interconnected. The connection between stages is manifested in the fact that internal investment $p_{k-1}^E$ is separated from the income of $(k-1)$-th stage and spent at the next $k$-th stage. Consequently, to solve the problem (1)-(8) we may
apply dynamic programming approach [Ve01]. To determine the recurrent relation connecting \((k-1)\)-th and \(k\)-th stages let us introduce the concept of conditionally optimal value of function \(F\) at \(k\)-th step \(F_k^{\Sigma}\):

\[
F_k^{\Sigma} = F_k(p_k^I, p_k^E, 0, x_k);
\]

\[
F_k^{\Sigma} = F_k^{\Sigma} + F_1(p_1^I, p_1^E, p_1^S, x_2);
\]

\[
\ldots.
\]

\[
F_N^{\Sigma} = F_{N-1}^{\Sigma} + F_N(p_N^I, p_N^E, p_N^S, x_N).
\]

Then the desired recurrent relation takes the form:

\[
F_k^{\Sigma} = F_{k-1}^{\Sigma} + F_k(p_k^I, p_k^E, p_k^S, x_k).
\] (12)

After those transformations the solution of the calculation problem for the optimal project financing schedule is an \(N\)-step process, wherein at each step a local optimization problem of the type is solved:

\[
F_k(p_k^I, p_k^E, p_k^S, x_k) \rightarrow \min,
\] (13)

with limitations (1)-(3), (7)-(8). This optimization problem is a four-variable linear programming problem to be solved by one of classic operations research methods. The solution obtained after \(N\)-step optimization is an advisory one and remains optimal at specified values of \(\alpha^I\), \(\alpha^E\) and \(\alpha^V\). By specifying various values of \(\alpha^I\), \(\alpha^E\) and \(\alpha^V\) we may shape a plurality of efficient solutions, each of them corresponding to an investment project financing schedule. Dependent on the situation on the capital market the program will select one of the available financing schedules. If a problem does not have any solution, it means that this project cannot be adequately financed under the specified conditions.

The algorithm diagram is represented on the fig. 3.
4 Software design

In order to perform an automated calculation of an optimal company funding schedule the software was developed. The program is implemented on Java in the IDE IntelliJ IDEA. For working with data MS SQL Server 2000 is used (the database diagram is represented on the fig. 4). The choice of the programming language was made in view of
outlook for using this software on different platforms. Though the DBMS MS SQL Server 2000 is platform-dependent, it was chosen because its drivers implement all necessary functionality provided by the JDBC specification. Yet no database specific features have been used in order to turn to usage of a cross-platform database or alternative way of data storing in future. Currently, there is considered a possibility of using the DBMS MySQL 5.0 or data storing in the XML format.

The general design of the software is represented by the fig. 5. The software has a standalone level for interaction with the database presented by the class DBContext. The “Database manager” component implements data management operations and converts entity data into business objects. The core of the computation module consists of the “Linear Optimization Module” and “Fuzzy Linear Optimization Module” components that implement algorithms of solving the linear programming problems and fuzzy linear programming problems accordingly. The main component is “GUI”, which implements the graphical user interface and the interaction of other modules of the program. Besides, the program includes the component “Logger” destined for logging service data (e.g. debugging messages, errors logging), and the component “Localization Manager” that customizes the language of UI according to the user’s OS settings (the English and Russian languages are currently supported).

<table>
<thead>
<tr>
<th>#</th>
<th>Entity name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Project</td>
<td>The entity describes the project and its main properties</td>
</tr>
<tr>
<td>2</td>
<td>Investor</td>
<td>Data about an external investor</td>
</tr>
</tbody>
</table>

Fig. 4 – Database diagram (physical view)
3 Enterprise | Data about a company (enterprise), which is engaged in the project execution
4 Model | An optimization mathematical model properties
5 Project_Plan | Schedules of inputs for the project execution and income from the project realization
6 Investor_To_Project | The entity describes the project specific data of an investor
7 Enterprise_To_Project | The entity describes the project specific data of a company (enterprise)
8 Enterprise_Investments_Plan | The schedule of the special investments provided by a company (enterprise)

Table 1 – Brief entities description

Fig. 5 – Components diagram

5 Working with the software and obtaining the results

The software has an user-friendly interface, it allows to store/load source data from the database, and the computation results are displayed in tables and chart views. User can also obtain a report on the preformed computation in HTML format. The key functions of the program are represented within the use-case diagram (fig. 6).
Fig. 6 – The key functions of the software

Fig. 7 shows an exemplary program window predestined for entry and editing of project financing process parameters. Here a user specifies the project, the contractor company, the investor and a set of specific parameters. The data on project, contractor and investor must be specified beforehand. When all data have been specified, the user may perform a calculation of the optimal financing scheme.
Fig. 7 – Editing window for financing process parameters.

An example of calculation results (in graphic form) as obtained with this software can be seen in Fig.8.
6 Conclusion

The developed information analytical system was applied as a demonstration model in
development of optimal financing scheme for project of new process lines at Kharkiv
Dairy Combined Works. The system is peculiar in accessibility of initial information for
calculation (those data being an integral part of any investment project business plan),
high velocity of optimization problem solution, offers possibility of study of various
financing sources and provides comfortable software handling, etc.

As this work omits the aspects of investment management due to selection of certain
taxation schemes, we propose to include a module containing various profit taxation
options for investors in the future. Further development of the work would also include a
calculation of optimal project financing scheme attracting several external financing
sources (such as direct investment, commercial credit, etc.)

Bibliography

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