Coalgebraic Theory of Büchi and Parity Automata: Fixed-Point Specifications, Categorically

Ichiro Hasuo
National Institute of Informatics, Japan
hasuo@acm.org
https://orcid.org/0000-0002-8300-4650

Abstract
Coalgebra is a categorical modeling of state-based dynamics. Final coalgebras – as categorical greatest fixed points – play a central role in the theory; somewhat analogously, most coalgebraic proof techniques have been devoted to greatest fixed-point properties such as safety and bisimilarity. In this tutorial, I introduce our recent coalgebraic framework that accommodates those fixed-point specifications which are not necessarily the greatest. It does so specifically by characterizing the accepted languages of Büchi and parity automata in categorical terms. We present two characterizations of accepted languages. The proof for their coincidence offers a unique categorical perspective of the correspondence between (logical) fixed-point specifications and the (combinatorial) parity acceptance condition.

2012 ACM Subject Classification Theory of computation → Automata over infinite objects

Keywords and phrases Coalgebra, category theory, fixed-point logic, automata, Büchi automata, parity automata

Digital Object Identifier 10.4230/LIPIcs.CONCUR.2018.5

Category Invited Tutorial

Funding Supported by ERATO HASUO Metamathematics for Systems Design Project (No. JP-MIER1603), JST; Grants-in-Aid No. 15KT0012 & 15K11984, JSPS; and the JSPS-INRIA Bilateral Joint Research Project “CRECOGI.”

Studies of automata, and state-based transition systems in general, have been shed a fresh categorical light in the 1990s by the theory of coalgebra [7, 5]. In the theory, a state-based dynamics is modeled by a coalgebra, that is, an arrow \( c: X \rightarrow FX \) in a category \( C \); and this simple modeling has produced numerous results that capture mathematical essences and provide general techniques.

Final coalgebras as “categorical greatest fixed points” play a central role in the theory of coalgebra. Somewhat analogously, most coalgebraic proof methods have focused on greatest fixed-point properties – a notable example being a span-based categorical characterization of bisimilarity.

In this tutorial, I will outline our recent results [10, 8] about how we can accommodate, in the theory of coalgebra, those fixed-point properties which are not necessarily the greatest. This takes the concrete form of characterizing the accepted languages of Büchi and parity automata in the language of category theory. Our framework, based on the so-called Kleisli approach to coalgebraic trace semantics [6, 4, 2, 1], is generic and covers both automata with nondeterministic and probabilistic branching. It covers both word and tree automata, too.
We present two characterizations of the accepted languages of Büchi and parity automata. The first one is called *logical* fixed points; it is formulated in terms of the order-enriched structure of the underlying Kleisli category (where the monad in question models branching type) [10]. The second one, called *categorical* fixed points, utilizes nested datatypes specified by a functor. The latter resembles repeated application of (co)free (co)monads. We exhibit a proof for the coincidence of the two characterizations. What arises through it is a categorical perspective of one of the key observations that underpin the recent developments in computer science – namely the fact that the *combinatorial* notion of parity acceptance condition represents *logical* specifications given by nested and alternating fixed points.

The tutorial is based on the speaker’s joint works with Corina Cîrstea, Bart Jacobs, Shunsuke Shimizu, Ana Sokolova, and Natsuki Urabe [2, 3, 8, 10]. A detailed account of the technical material of the tutorial will be given in a forthcoming paper [9].

---

**References**


